Investing Your Vote – On the Emergence of Small Parties

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Election thresholds in proportional election systems prevent small parties from entering parliament. Nevertheless, parties below the threshold often do get votes. We rationalize this behavior in a two-period model with aggregate uncertainty about whether a newly emerged party has sufficient support to pass the threshold. In equilibrium, passionate supporters of the new party vote for the party despite expecting the party to fail the threshold. Their votes can signal a strong backing in the population and increase the party's chances to enter parliament at the next election.

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1. Introduction

In parliamentary systems with proportional representation, the seat share of parties in parliament is approximately proportional to their vote share. In order to enter parliament, however, a party has to receive some minimal amount of votes, called the election threshold. An *implicit* threshold is present in any system and amounts to the minimum vote share necessary to get a single seat. The implicit threshold generally depends on the number of seats in parliament, but also on institutional details and the distribution of votes. Beyond that, many countries have a more restrictive *explicit* threshold. It typically ranges from 2% of votes in Israel and Denmark to 5% in countries like Belgium or Germany. Russia (7%) and Turkey (10%) have particularly high thresholds.

Rational voters supposedly cast their ballot only for parties that they anticipate to enter parliament. With respect to a majoritarian system, Duverger (1951, 1954) was the first to point out that votes are often concentrated on two dominant parties.[1] Downs (1957, p. 48) summarizes the reasoning as follows: "A rational voter first decides what party he believes will benefit him most; then he tries to estimate whether this party has any chance of winning." The same reasoning applies to proportional systems with election thresholds. However, in that case, the relevant question is whether a party enters parliament and, if so, whether it joins a governing coalition. "Falling short of such a threshold means that a vote for a party is 'wasted' or 'lost' because it does not count toward the distribution of seats in parliament" (Meffert and Gschwend, 2011, p. 3).

Despite this reasoning, small parties often do get a considerable amount of votes while failing the threshold. Extreme examples include the Russian legislative election in 1995, in which almost half of all votes were split among parties falling short of the threshold. Similarly, in the 2002 election in Turkey, about 45% of votes were unrepresented in parliament.[2]

Existing models of political competition typically neglect this issue. To fill this gap, we consider the following setting: Two incumbent parties split the majority of votes between themselves. A new party tries to enter parliament, but is expected to fall short of the threshold. Given this expectation, why should anyone vote for the new party?

Using a game-theoretic approach, we show that for some voters it is in fact strategically optimal to vote for a party that does not enter parliament at the current election. Rather than wasted, their vote is invested – trading-off influence at the current election against

[1] See Da Silva (2006) for the recent discussion of Duverger's Law in the political literature.

[2] References are provided by the BBC (www.bbc.co.uk/turkish/secim2002/election_results_en.shtml, retrieved March 2010) and by White, Wyman, and Oates (1997, Table 2), respectively.

future policy returns. On the other hand, some voters strategically vote for an incumbent party despite preferring the new party's platform.

Voters' preferences over policies depend on their type and there is aggregate uncertainty about the type distribution. Given their priors, citizens vote strategically to maximize their expected utility. Voting takes place in two consecutive periods. In the second period, voters update their beliefs about the type distribution, based on the first election result. This gives rise to (at least) two additional motives to vote for a party in period one. First, these votes may be a signal to other supporters of the small party to vote for the party in the next election, because its probability of entering parliament is higher under the revised prior than under the initial prior. Second, these votes may induce incumbent parties to change their platform due to updated beliefs about the number of voters that could be attracted by a given platform. In a related work, Castanheira (2003) focuses on the second motive. In our analysis, we explore the first one and take party positions as exogenously given. Notice that these two motives predict different voting patterns over time. In particular, adapting their platforms allows incumbent parties to keep entrant parties out of power; in the last period no voter supports the entrant parties. If the entrant party is successful in our model, its vote share is increasing over time and it might become part of the government.

We characterize an equilibrium in which supporters of the new party make their second-period ballot decision conditional on the results of the first election. In particular, they coordinate on the new party if the party's vote share has exceeded some endogenous threshold. Otherwise, the entrant party will receive no votes in the second period. Figure 1 outlines this *investing equilibrium*. Our findings rationalize a voting behavior that looks wasteful upon first sight, but turns out to be strategically optimal. Hence, there is no coordination failure in the sense of inconsistent beliefs about equilibrium play.

In the investing equilibrium, the first period serves as a costly coordination stage for supporters of the entrant party. Passionate voters reveal their type by voting for the entrant knowing that it will fail the election threshold. The costs arise from giving up influence on parliament's current composition.

Polls could offer a possibility to reveal types without these cost. Yet, the absence of such a cost is also a major drawback, as the signal becomes less credible. If polls were able to serve the signaling purpose nonetheless, the electoral patterns motivating our model are unlikely to occur. The costs of using the vote as a signal seem necessary in order to make the signal credible.

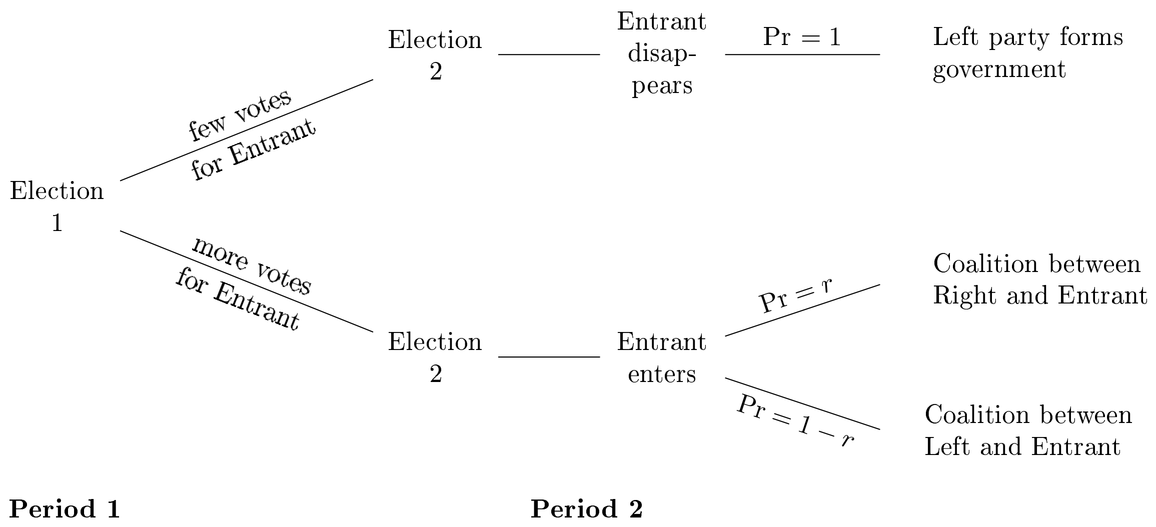

Figure 1: Three parties, Left, Right, and Entrant, compete in two consecutive elections. In period 1, the entrant party in equilibrium fails the election threshold. Yet, if the entrant's result in the first election is sufficiently strong, the party enters parliament in period 2. r denotes the fraction of supporters of the right party.

Our model is consistent with the paradigm of instrumental voting: our voters care exclusively about the policy impact of their vote. This is in line with empirical findings summarized in Section 3 suggesting that voters do use their votes strategically. An alternative explanation goes under the heading of 'expressive voting': By voting for a particular party, a voter might want to express certain opinions regardless of the effects on election outcomes and gain utility just from the act of doing so.[3] Our model suggests that voting behavior that looks expressive at first sight could well be instrumental in the long run.

The rest of this article is organized as follows. Section 2 provides empirical motivation. Section 3 discusses related literature. Then, Section 4 sets up the model and Section 5 details on coalition formation and policy choice. Section 6 gives the main results of the paper. Section 7 discusses sincere voting. We conclude in Section 8. All proofs are relegated to the Appendix.

[3]Hamlin and Jennings (2011) provide an introduction to this line of thought.

2. Empirical Motivation

In a survey two months before the 2009 German national elections, 6% of the respondents considered voting for the Pirate Party, which ran for the first time for national parliament. In addition, 58% reported not knowing the party.[4] The expected vote share for the Pirate Party was 2% according to polls, i.e., well below the German threshold of 5%. In the election, the Pirate Party received in fact 2% of the votes. Hence 2% of the electorate voted for the Pirate Party despite its low chance to enter parliament. On the other hand, the Pirate Party voters had reasons to believe that the number of supporters of the Pirate Party is in fact higher than 2%, namely 6% plus a potential fraction of people who did not know the party yet. We offer an explanation for why these 2% indeed had a strategic incentive to vote for the Pirate Party. Our model suggests that the early Pirate Party voters played an important role for the subsequent success of the Pirate Party in recent elections at state level.[5]

In Austria, a new right-wing party, the BZÖ, was founded in April 2005 by former members of the FPÖ, another right-wing party. According to polls for the general election 2006, the BZÖ was expected to receive a vote share of just around 4%, which is the election threshold in Austria.[6] So there was high uncertainty on the party's probability to enter parliament. With an election result of 4.1%, the BZÖ did succeed after all.[7] Two years later, in the general election of 2008, the BZÖ more than doubled its vote share to 10.7%. It seems likely that a significant fraction of voters preferred the BZÖ already in 2006, but refrained from voting for the BZÖ for fear of wasting their vote, as the BZÖ might have missed the election threshold. In 2008, these voters went for the BZÖ, as there was less uncertainty about the BZÖ entering parliament. Our model can explain such a jump in the vote share without a change in voters' preferences.[8]

The German Green Party was founded in 1980 and failed to pass the 5% threshold in the national election of the same year. They obtained a significant fraction of 1.5% of the votes, though. In the 1983 national election, they successfully passed the threshold.

[4] The survey was conducted by TNS Emnid for Cicero, a German magazine, and published on the 23rd of July 2009.

[5] The Pirates received 8.9% in the state of Berlin in 2011, and 7.4% in Saarland, another German state, in 2012. They entered parliament in both states. For a reference, see www.dw.de/cdu-holds-its-own-in-saarland/a-15837387-1, retrieved May 2012.

[6] A party enters parliament in Austria, if it wins more than 4% of the votes or if it crosses a particular threshold of the valid votes in one of the 43 regional electoral districts. See also Solsten (1994).

[7] Meffert and Gschwend (2010) provide extended discussion of the 2006 election.

[8] Notice, though, that our analysis has to be changed slightly to a fit the BZÖ case. We present a more clear-cut version in which, along the equilibrium path, the small party actually has no chance of entering parliament in the first period.

We argue that this success would not have been possible, had they received insufficiently many votes in 1980. Hence, their 1980 voters did not waste their votes but effectively invested them.

On the other hand, our model predicts that if a new party falls short of an endogenous threshold in the first election, its support disappears. There are numerous examples of such developments. Consider, for instance, Proud of the Netherlands, a party founded in 2009. Although early polls showed the party to win up to 10%, it gained a vote share of only 0.6% or 52,937 votes in its first general election in 2010. This is slightly below the Dutch election threshold of 1/150 to enter parliament. Although the party merged with another one to compete in the 2012 general election, its election results decreased significantly to 0.1% or 7,363 votes. Another example is Action for Democratic Progress, a party founded in 1968 in Germany to protest against the German Emergency Acts. The party participated in the federal election of 1969, but gained a vote share of only 0.6%, i.e., less than 200,000 votes. The very same year, the party was dissolved.[9]

3. Related Literature

We contribute to the literature on communicative voting. A recent contribution in this area is Aytimur, Boukouras, and Schwager (2013), who focus on election turnouts. Similar to our results, voting can signal unobserved characteristics. Our research question is different, though. We consider a model which specifically captures the uncertainty about the support of a new party. Our setting fits the multi-party election systems with proportional representation which are typical for continental Europe. Parties have to pass an election threshold to enter parliament. These thresholds create non-trivial voting incentives, which are at the heart of our analyses. Related contributions differ not only with respect to the relevant setting but also with respect to the modeling of uncertainty and the addressees of the signal.

Razin (2003) employs a one-period model with common values, in which election results are informative for the legislature about some shock affecting the country. This allows candidates better to adapt policies to the state of the world. In our model, the signal is aimed at the electorate rather than the legislature. We analyze competition between three parties as opposed to two, as in Razin, 2003. Therefore we can address

[9]See: www.verkiezingsuitslagen.nl/Na1918/Verkiezingsuitslagen.aspx?VerkiezingsTypeId=1, www.dutchnews.nl/news/archives/2012/06/new_dpkp_party_marks_a_turning_1.php, www.bundeswahlleiter.de/de/bundestagswahlen/fruehere_bundestagswahlen/btw1969.html; all retrieved June 20, 2013.

the coordination problem of voters that arises if each party needs some minimal support to have any effect on policy.

Piketty (2000) allows voters to signal their information about the state of the world by electing specific candidates. They want to influence future voting behavior. The two-period horizon is similar to our paper. Yet, in Piketty's model, voters are uncertain about the optimal policy and the candidate they prefer. In our model, every voter knows with certainty which party she prefers, but is uncertain about the preferences of other citizens. The problem of coordination is not prominent in Piketty's paper, because only two alternatives are offered in each of the elections.[10]

In the model proposed by Castanheira (2003), the position of the median voter on a one-dimensional policy space is unknown. Four parties compete under majoritarian rule in two consecutive elections. After the first election, voters update their beliefs about the true position of the median voter. Votes for an extreme party signal that the median voter is at a more extreme position. Given updated beliefs, parties might reallocate their platforms for the next election. Voting for "losers" is a rational strategic choice so as to induce parties to change their policy offers.

In our contribution, voters have incentives to vote for "losers" as well; however, they do not send a signal to parties but rather to other voters. The question is not whether parties adjust their platforms, but whether they have sufficient support to enter parliament.[11] Ex ante there is no clear answer to this question, because there is uncertainty about voters' types.

In Meirowitz and Shotts (2009), there is uncertainty about voters' preferences, as well. Two candidates compete in two consecutive elections. After the first election beliefs are updated and candidates might reallocate their platforms for the next election. They show that for large electorates the signaling effect dominates any pivot effects. The reason is that the probability of being pivotal converges to 0 very fast while every vote can signal at least some information.

In all of the above papers, voters' types are independently and identically distributed. We assume aggregate uncertainty in the sense that the joint distribution of voters' types is random. This distribution is drawn according to a commonly known second-order distribution. In contrast to the first approach, this aggregate uncertainty does not

[10] Two candidates compete in the first election and the winner is challenged by a third candidate in the second election.

[11] We take it as given that three parties compete, but presume one to be new and small. The question whether a new party emerges at all is addressed by Palfrey (1984). In his model, incumbent parties may anticipate entry of new parties and adopt their policies in advance in order to prevent entry.

depend on the number of voters. In particular, our approach ensures uncertainty even for a large number of voters. For the same reason, Bierbrauer and Hellwig (2011) use this approach in a public economics setting.

Coalition formation plays a central role in proportional elections. In contrast to the aforementioned contributions, we include coalitional bargaining in our model. Therefore, sub-majority parties in parliament can also influence policy. We build upon the proto-coalition bargaining model of Baron and Diermeier (2001).[12] Their two-dimensional policy space is able to capture a situation that is typical for the emergence of a new party. Namely, incumbent parties differ with regard to traditional issues, but have similar views on a new issue such that parliament does not reflect different opinions on that new issue. A new party emerges for the very reason to represent this new issue or some hitherto unrepresented opinion. Examples include green parties which promoted environmental protection or the Pirate Party with its focus on digital rights.

We adapt the Baron and Diermeier (2001) setting in various respects. First, we assume that the distribution of voters' preferences is uncertain. Second, we include an intensity parameter in voters' preferences. Third, we introduce a second election period. This gives rise to communicative voting motives that drive our model. Furthermore, we make specific assumptions about the peaks of the parties and the distribution of voters in order to capture the particular situation that makes the election threshold interesting. We also slightly adapt the bargaining procedure.

In terms of empirical literature, we relate to studies that investigate strategic voting.[13] Bargsted and Kedar (2009) examine elections in Israel in 2006. They find that the set of perceived coalition possibilities had a profound influence on voter ballots in the sense that it induced non-sincere voting, i.e., voting for parties other than the most preferred one. Baron and Diermeier (2001, p. 934) provide another example for strategic voting behavior in which supporters of a larger party vote for a small party to make them enter parliament. Similarly, Shikano, Herrmann, and Thurner (2009, p. 634) "show that voters' preferences, rather than mapping directly into party choice, are affected by

[12] Further prominent contributions on coalitional bargaining with a focus on political economy include Baron and Ferejohn (1989) and Austen-Smith and Banks (1988). They do not match our setting as well as Baron and Diermeier (2001), though. Baron and Ferejohn (1989) concentrate on the bargaining between legislators who serve their own district, but do not include a general election stage. Austen-Smith and Banks (1988) propose a more complete model of the political process. Similar to our paper, they consider three parties, strategic voters, and endogenous coalition formation. However, they focus on parties' platform choice, whereas our main attention is on voters' decisions. Furthermore, their one-dimensional policy space is not rich enough for our purposes.

[13] Meffert and Gschwend (2011, pp. 2-3) provide a long list of evidence for strategic voting in different countries.

their expectations on small parties' re-entry chances." Meffert and Gschwend (2011) conduct a laboratory experiment embedded in a state-level election in Germany in 2006. Participants in their study were able to form meaningful expectations about election outcomes and possible coalitions. McCuen and Morton (2010) also show strategic voting under proportional representation in a laboratory experiment.

Sobbrio and Navarra (2010) explicitly investigate communicative motives in voting. For a sample of 14 European countries, they find that voting for a "sure loser" is associated with higher education. They see this as evidence that voters consider more than one election period in their voting decision.

Finally, Kricheli, Livne, and Magaloni (2011) propose a model of civil protest against repressive regimes. In a first period, a small group of protesters take to the streets to signal their type. If their protest reaches a critical size, it triggers a mass protest able to overthrow the dictator. Kricheli, Livne, and Magaloni (2011) back their theory with an empirical analysis, according to which more repressive regimes tend to be more stable, but protests that do occur pose a higher threat to them.[14]

4. Model

Society consists of n citizens and has to make a two-dimensional policy choice

$$x = (a, b) \in X = [0, 1] \times [0, 1].$$

Three parties, called *Left* (L), *Right* (R), and *Entrant* (E) compete for proportional representation in parliament. In the ballot, each citizen casts a vote for one of the parties. Parties receiving less than $\tau \in \mathbb{N}$ votes do not get any seats in parliament. The seat share of a party that enters parliament equals its vote share corrected for votes unrepresented in parliament.[15] The parties entering parliament bargain to form a government (coalition) which needs to hold at least a simple majority, i.e., half the seats. Government chooses policy. The model consists of two consecutive election periods. Fundamentals stay constant over time, but beliefs may change.

Our analysis focuses on voters' ballot decisions rather than on the question whether

[14] The more repressive a regime, the higher the costs of failed protest. Initial protest is then less likely to occur but constitutes a stronger signal.

[15] For the sake of tractability, we abstract from non-divisibility issues of seats. Hence, a party's seat share equals the number of its votes divided by the sum of votes of those parties that enter parliament.

to vote at all.[16] Voters in our model have a strictly positive benefit from voting, so our results are robust if voting costs are sufficiently small. Alternatively, consider voters who have already made their way to the voting booth. Now they just need to decide which party to vote for. This is the decision we analyze.

4.1. Parties

Each party $J \in \{L, R, E\}$ is characterized by its peak (a_J, b_J). Each period, parties gain utility $u^J(x)$ from policy x, and might receive benefits s_J from holding office, like positions in the cabinet or seats on the boards of public companies.[17]

$$U^J(x, s_J) = u^J(x) + s_J = -(a - a_J)^2 - (b - b_J)^2 + s_J.$$

The total spoils of holding office are $S > 0$. The parties split these spoils among them such that $\sum_J s_J = S$. In addition, $s_J \geq 0$ if J is not in government.[18] Note that the specification allows for unlimited side payments among members of the government.

The first dimension of the policy decision can be interpreted as a traditional issue (left vs. right) over which the incumbent parties L and R struggle. L prefers $a = 0$, hence $a_L = 0$, while $a_R = 1$. The second policy dimension, b, is new in the sense that it has not been much of an issue in the past. This is reflected in identical peaks, namely $b_L = b_R = 0$. The entrant party E disagrees on the new issue and likes to initiate a change in b by offering a political alternative, thus $b_E = 1$. Let it emerge from the left spectrum such that $a_E = 0$.[19] To summarize, we have

$$(a_L, b_L) = (0, 0), \qquad (a_R, b_R) = (1, 0), \qquad (a_E, b_E) = (0, 1).$$

Parties cannot make binding commitments. After each election, bargaining among the parties represented in parliament determines the policy in the current period. Section 5 provides details about bargaining procedures and policy choice.

[16] The paradox of not voting is the focus of a related signaling-voting model recently proposed by Aytimur, Boukouras, and Schwager (2013).

[17] For a detailed discussion of s_J see Baron and Diermeier (2001, p. 935).

[18] Parties outside the governing coalition cannot be forced by coalition members to pay the ruling parties. Hence a negative value of s_J is allowed only for coalitions members.

[19] Our results would be the same for a symmetric variation of the model in which the new party emerged from the right spectrum. The new party could also position itself somewhere between left and right. This would require a richer model in terms of voter preferences, though, and would complicate the model without enhancing the insights with respect to the investment equilibrium.

4.2. Voters

Voters have preferences on policies. They differ, in particular, with respect to the salience of the two policy dimensions. A voter of type $\theta^i \equiv (a^i, b^i, \alpha^i) \in \Theta$ has the per-period utility function

$$u(x; \theta^i) = -\alpha^i(a - a^i)^2 - (1 - \alpha^i)(b - b^i)^2,$$

where (a^i, b^i) corresponds to the peak with respect to the two policy dimensions and $\alpha^i \in (0, 1)$ describes the relative salience of the two dimensions from the perspective of the voter. There is a common discount factor $\delta > 0$. The second period can alternatively be interpreted as some long-run steady state spanning multiple periods. In that case, δ serves as a weighting factor and might be above 1.

To keep the model as simple as possible, we consider only four different types of voters: *left* (θ^L), *right* (θ^R), *passionate* (θ^P) and *moderate* (θ^M), with

$$\theta^L = (0, 0, 1/2), \qquad \theta^R = (1, 0, 1/2), \qquad \theta^P = (0, 1, \alpha^P), \qquad \theta^M = (0, 1, \alpha^M). \tag{1}$$

Left and right types mirror the preferences of the respective parties. Traditional and new issues are of equal importance to them. The supporters of the entrant party share its peak, but differ in their assessment of the importance of the new policy dimension b. Some of them are more passionate for the new issue, while others have a stronger attachment to the traditional issue and are hence more moderate in their support for the entrant. Formally, we have $\alpha^P < \alpha^M$. Figure 2 depicts the preferences of the four types.

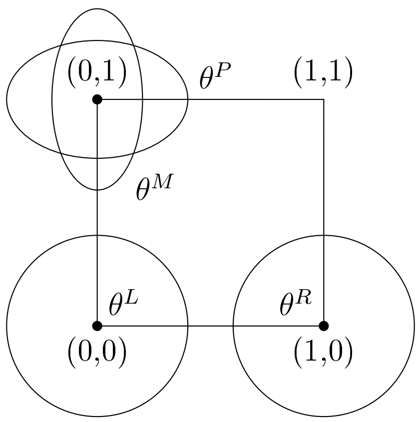

Figure 2: Black dots depict the peaks of the four types. Circles and ellipses indicate typical indifference curves. Notice that passionates and moderates share the same peak, but differ with respect to the relative importance of the two issues.

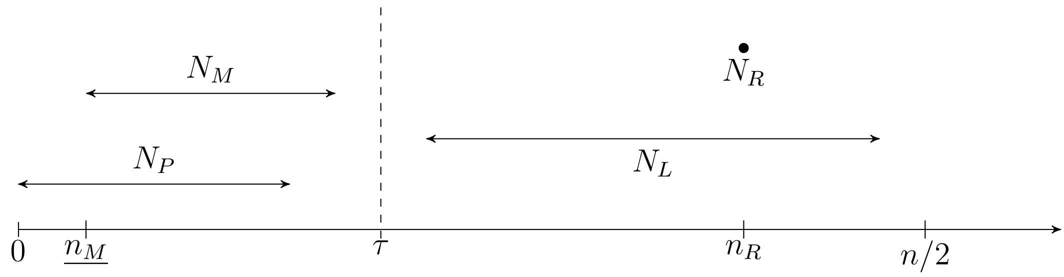

Figure 3: The figure illustrates Assumption 1. The number of right types, N_R, is know to equal n_R, strictly between the election threshold τ and an outright majority. The numbers of passionates, N_P, moderates, N_M, and left types, N_L, are random variables. Their support is known to be in the depicted intervals.

The distribution of voter types is unknown. In contrast to previous literature, in which voters' types are determined by *iid* draws, the whole distribution of voters' types is drawn at once from a joint distribution. This creates aggregate uncertainty, even for large electorates. Let N_j be the random number of voters of type θ^j. We denote a distribution of voters' types by $N = (N_R, N_L, N_M, N_P)$ with $N_R + N_L + N_M + N_P = n$. N is randomly drawn according to a common prior P. Through the following assumption concerning the support of P, we restrict attention to the specific scenarios addressed in the introduction.

Assumption 1. *The prior P is such that there are constants $n_R \in (\tau, n/2)$ and $\underline{n_M} \geq 3$, and*

$$\Pr(N_R = n_R) = 1, \qquad \Pr(N_L = y) \begin{cases} > 0 & \textit{if } \tau < y < n/2 \\ = 0 & \textit{otherwise} \end{cases}$$

$$\Pr(N_M = y) \begin{cases} > 0 & \textit{if } \underline{n_M} \leq y < \tau \\ = 0 & \textit{otherwise} \end{cases}, \qquad \Pr(N_P = y) \begin{cases} > 0 & \textit{if } y < \tau - 2 \\ = 0 & \textit{otherwise} \end{cases}$$

The assumption, illustrated in Figure 3, guarantees that the number of voters on the right wing, $N_R = n_r$, and on the left wing, $N_L + N_M + N_P = n - n_R$, are known, e.g., due to previous (non-modeled) elections. The distribution within the left wing is uncertain, though. None of the two groups of entrant supporters is able to meet the election threshold on its own. Jointly they have a chance to do so. The specification allows for a correlation of N_P and N_M, but we do not require it. The number of moderate types has a commonly known lower limit of $\underline{n_M}$. Finally, both L and R have more than

τ supporters but less than the absolute majority of voters. Notice that Assumption 1 restricts the support, but not the probabilities.

4.3. The Election Game

There are two consecutive elections. Each consists of a ballot, in which all voters cast their vote, and a subsequent bargaining stage, in which parties determine policy. The exact timing is a follows:

$t = 0$	N is drawn according to prior P, voters learn their own type θ^i.
$t = 1$	Voting takes place. – Parties above the election threshold enter parliament. – Coalition bargaining determines the governing coalition. – The governing coalition implements policy, payoffs are realized.
$t = 1.5$	Voters observe the result of the first election and update their beliefs.
$t = 2$	Repetition of $t = 1$.

Notice that nature only turns once, at the beginning of the game; the preference distribution remains constant during the whole game but is not public information. Therefore, voters will want to update their beliefs using the first election result in order to make better-informed decisions in period two.

We detail on the coalitional bargaining stage in the Section 5. For now, suppose that policy in period t is given by some random variable X^t. At each election, voters (correctly) anticipate the bargaining outcome. We consider the following *election game*: Let v_J^t be the number of votes for party J in period t, and let $V^t := \{(v_R^t, v_L^t, v_E^t) \in \mathbb{N}_0^3 | \sum_J v_J^t = n\}$ be the set of all possible results in period t. Recall that Θ is the type set. Behavior of voter i is given by a pure strategy $\sigma_i = (\sigma_i^1, \sigma_i^2)$, which specifies voting decisions $\sigma_i^1 \colon \Theta \to \{R, L, E\}$ for period one, and $\sigma_i^2 \colon (\Theta, V^1) \to \{R, L, E\}$ for period two. Each strategy profile $\sigma := \{\sigma_1, \ldots, \sigma_i, \ldots, \sigma_n\}$ induces distributions of votes, $v^1(\sigma)$ and $v^2(\sigma)$, which are random due to the uncertainty about N.[20] Parties' vote shares determine their seat shares, accounting for those parties failing the election threshold. Policy is endogenously derived from parties' seat shares, as detailed in Section 5. Thus there is a mapping $X^t(\sigma)$ from strategy profiles into the policy space. The mapping is random due to the randomness in types and the nature of the bargaining process.

[20] Notice that our strategy definition is restrictive. In general, the second-period voting decision could depend not only on voters' type and the previous election outcome but also on the parties' action and first-period policy. Our main results, however, remain valid if we allow for these general strategies.

Anticipating policy choice $X^t(\sigma)$, voter i chooses strategy σ_i in order to maximize expected utility

$$U(\sigma_i, \sigma_{-i}; \theta^i) = \mathbb{E}[u(X^1(\sigma_i, \sigma_{-i}); \theta^i)|\theta^i] + \delta\mathbb{E}[u(X^2(\sigma_i, \sigma_{-i}); \theta^i)|\theta^i], \tag{2}$$

given the strategy profile σ_{-i} of all other voters. We look for a Perfect Bayesian Equilibrium of the election game.[21]

5. Coalition Bargaining

After each election, the parties represented in parliament determine policy. If a party has a seat share of more than 50%, this party alone decides about policy and receives all spoils of office. If no party has such a majority, a coalition is necessary to form government. Then one party is randomly selected to be the *proposer*. The selection probability is equal to the seat share. The proposer makes a take-it-or-leave-it offer to a subset of parties that (including the proposer) holds at least 50% of seats in the parliament. The offer specifies a policy and a distribution of the spoils of office s_J. All members of the proposed coalition have to accept the proposal; if at least one of them rejects, a default policy is implemented and no party gains any spoils of office.[22]

The default policy is linked to the (non-modeled) past. As we model an old struggle over the traditional issue only, a natural candidate for the default policy is the one who emerges as a compromise between left and right. We thus assume that the default policy x_d is equal to $(1/2, 0)$. The results do not depend on this specific value, but are robust to some perturbations, as shown in Lemma 2 below.

Due to the quasi-linearity of parties' preferences in spoils of office, the policy implemented by some coalition depends only on the preferences of its members, not on the

[21] The solution concept allows only for uncertainty about the distribution of policy preferences in the population. There is no strategic uncertainty, because voters know which strategy is played by other voters. It is embedded in the equilibrium concept that every voter correctly anticipates others' strategies. There is no coordination failure in the sense that some voters believe equilibrium A is played, while others play equilibrium B. By using Perfect Bayesian Equilibrium, we highlight the effects that arise from the unknown type distribution N.

[22] This modeling of a coalitional bargaining process within a general election model was proposed by Baron and Diermeier (2001). We make the simplification that coalitional bargaining only takes place if necessary, i.e., if no party has more than 50% of seats. In terms of empirical evidence, the Baron-Diermeier model is supported by Diermeier and Merlo (2004), who explore data of coalition formation in 11 European countries spanning the years 1945–1997. They find that a selection probability corresponding to seat shares fits the data quite well and outperforms selection according to the seat share ranking.

default policy nor the relative seat shares within the coalition.[23] The default policy does determine how the spoils of office are distributed between coalition members. Those are not relevant for voters, though.

What is important to voters is that for each possible coalition there is a unique induced policy. Therefore, the ballot affects the probability distribution over the set of possible coalitions. In fact, no more than four policies are possible as outcomes of the bargaining game when we restrict attention to elections in which the Entrant misses an outright majority.

Lemma 1. *Let the seat share of party E be below 50%. Then in any equilibrium and in any period the coalition resulting from coalitional bargaining is* $\{L\}$, $\{L, E\}$, $\{R\}$ *or* $\{R, E\}$ *and the respective policy is* x_L, x_{LE}, x_R *or* x_{RE} *with*

$$x_L = (0, 0), \qquad x_{LE} = (0, 1/2)\,, \qquad x_R = (1, 0), \qquad x_{RE} = (1/2, 1/2)\,.$$

Furthermore, if no party has an outright majority of more than 50%, it is optimal for R to propose coalition $\{R, E\}$ *and for L and E to propose* $\{L, E\}$.

With respect to conditional probabilities of equilibrium policies, Lemma 1 implies the following. If E fails to enter, x_L occurs with $\Pr[v_L^t > v_R^t] + \Pr[v_L^t = v_R^t]/2$ while x_R occurs with complementary probability. Neither x_{LE} nor x_{RE} occurs. If all three parties enter parliament and each receives less than 50% of votes, then x_{LE} occurs with probability $[v_L^t(\sigma) + v_E^t(\sigma)]/n$ and x_{RE} occurs with complementary probability $v_R^t(\sigma)/n$.

Figure 4 illustrates the location of possible equilibrium policies. These do not depend on the seat shares. A coalition of, say, Right and Entrant will always implement x_{RE}, independently of their relative size. The seat share does determine the likelihood of the respective policies, though. The default policy is relevant for the proposer's decision which coalition to opt for. The Right Party, for instance, proposes to the Entrant because the default would be very bad for the Entrant and thus the Entrant is willing to accept a high side payment to Right in order to prevent the status quo. Right can thus secure a high overall payoff. Its optimal proposal could change if the default policy

[23] This is true also for more complex bargaining structures where parties can make counter-proposals. In this case, reservation utilities of parties depend on their endogenous continuation values rather than default policies. Party preferences over spoils and policies ensure that the policy implemented by coalition C does not depend on the particular bargaining process. For further discussion, see Baron and Diermeier (2001).

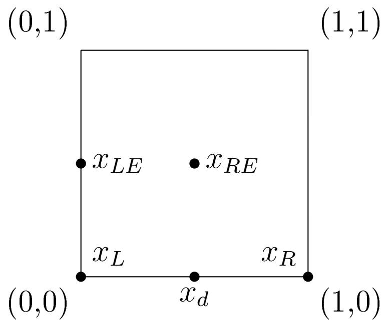

Figure 4: The coalitional bargaining process has a limited number of policy outcomes. There will either be a single-party government of Left or Right implementing x_L or x_R, or a two-party coalition including the Entrant implementing x_{LE} or x_{RE}.

was located elsewhere. But, according to the following result, Lemma 1 is in fact robust to some perturbation in x_d.[24]

Lemma 2. *Lemma 1 is valid for all default policies $x_d = (a_d, b_d)$ with*

$$x_d \in \{(a_d, b_d) \in [0,1]^2 \mid b_d \leq 1/4 \leq a_d \text{ and } a_d^2 + b_d^2 \leq 1/3\}.$$

Having analyzed party behavior after an election, we now turn to the voting decision and to the question why a voter should support a small party that is expected to fail at the election threshold.

6. The Investing Equilibrium

Put yourself in the situation of a voter in the booth. Left and right types expect their preferred parties to enter parliament and can vote sincerely. Supporters of the small party have a harder time. They like E to enter parliament which requires voting for E. On the other hand, if the Entrant fails the election threshold, a vote for the Entrant is wasteful, and voters would be better off voting for the left party. If the probability of E failing the threshold is sufficiently high, voting for them is thus never optimal in a one-shot game. The existence of a second period changes the game, though. Suppose it is certain that E will make it into parliament in $t = 2$ if passionate and moderates

[24]On the other hand, if the default policy is outside the set given in Lemma 2, the parties propose different coalitions: If $b_d > 1/4$, then R proposes a coalition with L, as the default policy is very bad for L in this case. If $a_d < 1/4$, then E makes R an offer, as the default policy puts R at disadvantage. Finally, if $a_d^2 + b_d^2 > 1/3$, R proposes a grand coalition of all three parties.

coordinate on voting for them. Then voting E is optimal in $t = 2$ and all supporters of the entrant are quite happy with the policy outcome. Call this the *entrance scenario*.

Now suppose voting in period one affects the probability of the entrance scenario. This additional incentive could induce supporters of the entrant to vote for the entrant even though this is wasteful in the short run. We show that this is indeed the case: passionate types vote for the entrant in the first period, actually knowing for sure that E will miss the threshold. By using their vote as a signal, they change the beliefs about the type distribution and foster the entrance scenario in the second period. Moderate types choose their second-best alternative in the first period and only jump the entrant bandwagon if it promises success in period two. To simplify the exposition of the investing equilibrium, we impose the following assumption on the prior P.

Assumption 2. *Voters of type θ^R are rather pivotal in the decision between R and L than in the decision whether E hits the equilibrium threshold. In particular,*

$$2\Pr\left[N_L + N_M \in \{n_R, n_R - 1\} | \theta^R\right] > \delta \Pr\left[N_P = \tau - \underline{n_M} | \theta^R\right] \left(3\frac{n_R}{n} - 1\right).$$

There is an outcome-equivalent equilibrium which works without Assumption 2, though (see Proposition 2). Also, for $n_R/n \leq 1/3$, the assumption is trivially satisfied.

Proposition 1. *For every prior P consistent with Assumptions 1 and 2, the following strategies constitute a Perfect Bayesian equilibrium if α^P is sufficiently small and α^M is sufficiently large, as specified by conditions* (4) *and* (5) *in the proof.*

- *Left and right types vote for their respective party in both periods.*
- *Passionate types vote for E in the first period, while moderate types vote for L. In the second period, they both vote for E if the first-period votes of E, v_E^1, meet a threshold e, defined as*

$$e = \tau + 1 - \underline{n_M}$$

  Otherwise all of them vote for L.

On the equilibrium path, the entrant party gets a positive vote share that is below the election threshold in the first election. It is common knowledge that it will not enter parliament. Nevertheless, passionate voters have an incentive to vote for the entrant in order to increase the probability of entry in the next election. In the second period, the new party either receives no votes at all or gets sufficient votes to enter parliament. While the entrant attracts only passionate supporters in the first period, once it has established itself its electorate becomes more moderate.

Corollary 1. *In the investing equilibrium, the ex-ante probability that the new party E enters parliament is* $\Pr(N_P \geq \tau + 1 - \underline{n_M})$*, the probability of sufficient passionate supporters.*

After the first election, voters believe that the number of passionate voters equals v_E^1, the votes for the entrant party. Thus, the entrant has sufficient support to meet the threshold τ if $v_E^1 \geq \tau - \underline{n_M}$. Yet, the equilibrium threshold e departs from that value as $+1$ is added to the threshold. This guarantees entry (conditional on meeting e) even if a non-passionate voter pretends to be a passionate type by voting E in $t = 1$. As a consequence, the in-equilibrium incentives of passionate and moderate voters have an identical structure. For both types, voting L instead of E yields a short-term benefit of

$$\Pr[pivotal|\theta^i] \cdot \big(u(x_L;\theta^i) - u(x_R;\theta^i)\big)$$

as voting L increases the chances of L rather than R building the first-period government. In the long run, however, voting E rather than L creates an additional value[25] of

$$\delta \Pr[pivotal|\theta^i] \cdot \left(\frac{n_R u(x_{RE};\theta^i) + (n - n_R) u(x_{LE};\theta^i)}{n} - u(x_L;\theta^i) \right),$$

as voting for E in period 1 increases the entrant party's probability to enter parliament in period 2. While the probabilities and the gains from changing the election outcome differ for the two types of entrant supporters, the basic trade-off between short- and long-run effects is the same. This analogy requires adding $+1$ to the endogenous threshold. Otherwise, there is a chance that a moderate voter voting for E triggers coordination for E in a state in which entrant supporters fall short of the threshold τ. This results in R having an outright majority and implementing x_R, the worst outcome for entrant supporters. The $+1$ excludes this case for individual deviations.

We call a voter 'passionate' if the long-run incentives outweigh the short-run benefits, and 'moderate' otherwise. While both effects are small, as they depend on the probability of being pivotal, they are both on the same scale.[26]

Robustness

It is possible to describe the investing equilibrium without Assumption 2.

[25] To be precise, voting for E has a long-run benefit for entrant supporter i only if $\alpha^i < 3/(n_R/n + 3)$. Otherwise, this voter cares so much about the traditional dimension that she prefers the certain alternative x_L over the lottery with x_{LE} and x_{RE}.

[26] We thank Daniel Krämer for bringing that point to our attention.

Proposition 2. *For every prior P consistent with Assumption 1, the following strategies constitute a Perfect Bayesian equilibrium if α^P is sufficiently small and α^M is sufficiently large, as specified by conditions* (4) *and* (5) *in the proof of Proposition 1.*

- *Left and right types vote for their respective party in both periods.*
- *Passionate types vote for E in the first period, while moderate types vote for L. In the second period, they both vote for E if the first period votes of E, v_E^1, meet a threshold e, defined as*

$$e = \begin{cases} \tau + 1 - \underline{n_M} + \max\{0, n_R - v_R^1\}, & \text{if } n_R/n > 1/3 \\ \tau + 1 - \underline{n_M}, & \text{if } n_R/n \leq 1/3. \end{cases} \tag{3}$$

  Otherwise all of them vote for L.

For $n_R/n > 1/3$, right types favor entry of the entrant party over an absolute majority of the left party. Thus, they might want to deviate from the equilibrium path by voting for E. To guarantee that such a deviation is not profitable, the endogenous threshold increases if less than n_R votes for R are observed in $t = 1$. On the other hand, if $n_R/n \leq 1/3$, then right types prefer an absolute left majority over the entrance of E. If they could raise the threshold by voting L (instead of R), they might want to do so to decrease chances of entrance of E in $t = 2$. To eliminate this incentive, the threshold is independent of v_R^1 in the case of $n_R/n \leq 1/3$.

So far, we concentrated on one particular equilibrium. The investing equilibrium serves our goal of explaining voting patterns as described in our motivation. In the following, we discuss another natural equilibrium candidate.

7. Sincere Voting

Voting is *sincere* if a voter chooses to vote for the party that matches the voter's preferences most closely. With two alternatives, sincere voting typically is a weakly dominant strategy, which makes it a prominent candidate for predicting behavior. Indeed, many political models take sincere voting for granted. With more than two alternatives, however, there is typically no dominant strategy, not even in the weak sense. This is true for our model as well. The question whether sincere voting is strategically optimal is far from trivial. We present a negative result in the following proposition.

Proposition 3. *For any prior consistent with Assumptions 1, sincere voting is not an equilibrium of the game.*

The intuition is that voters learn the distribution of types after the first election. Therefore, they can increase their utility by making their voting decision dependent on this information. In particular, a moderate voter can deviate to L instead of E in the second period, if she knows that E cannot enter parliament as the number of moderate and passionate types in the population is too low.

This points to a second equilibrium candidate: In the first period, everyone votes sincerely. But if voters learn that E cannot enter parliament, moderate and passionate types vote for L instead of E in the second period. Otherwise, there is sincere voting in the second period also. We call this strategy *conditionally sincere voting.* Notice that this strategy – at least in the second period – is similar to the strategies used in the investing equilibrium. Yet, we show that conditionally sincere voting is not always optimal, even under a condition in which the investing equilibrium exists. To see why, consider a moderate voter switching to L in the first period. Define P^L as the change in probabilities of L forming the government caused by such behavior.[27]

$$\begin{aligned} P^L =& \frac{1}{2}\Pr\left[N_P + N_M < \tau \wedge N_L \in \{n_R - 1, n_R\}\right] + \Pr\left[N_P + N_M = \tau \wedge N_L \geq n_R\right] + \\ &+ \frac{1}{2}\Pr\left[N_P + N_M = \tau \wedge N_L = n_R - 1\right]. \end{aligned}$$

There are three terms. First, the new party does not pass the threshold anyhow and does not enter parliament. Then the voter could be pivotal in the decision between L and R. The second and third terms capture the case of the new party missing the threshold exactly for the one voter.

Assumption 3. *A moderate type is rather pivotal in the decision between R and L than in the decision whether E enters parliament. In particular,*

$$4P^L > \left(4 - \frac{n_R}{n}(1+\delta)\right)\Pr\left[N_P + N_M = \tau \middle|\, \theta^M\right].$$

Given this assumption, a voter of type θ^M has a profitable deviation if she is sufficiently moderate.

[27] All probabilities are conditional on $\theta^i = \theta^M$. The dependence is suppressed for notational convenience.

Proposition 4. *For any prior P consistent with Assumptions 1 and 3, conditionally sincere voting is not an equilibrium of the game if α^M is sufficiently large, as specified by equation* (6) *in the proof.*

In the corresponding one-period model, sincere voting is no equilibrium either.

Corollary 2. *Consider a one-shot version of the model with only one election period. Sincere voting is not an equilibrium of the game for any prior consistent with Assumptions 1 and 3 if α^M is sufficiently large, as specified by equation* (6) *with $\delta = 0$.*

Finally, we conclude that the result of Proposition 4 is consistent with previous results.

Lemma 3. *For every prior consistent with Assumptions 1 and 3, there are α^P and α^M such that condition* (6) *in the proof of Proposition 4 and conditions* (4) *and* (5) *in the proof of Proposition 1 are satisfied.*

Therefore, for some parameter values the investing equilibrium exists, while neither sincere voting nor conditionally sincere voting is an equilibrium. Then the investing equilibrium can explain the observed voting pattern while sincere voting cannot.

8. Conclusion

In this paper, we set up a model in which voters coordinate their voting intentions by using their votes as a signal. This allows others to learn about the distribution of preferences in the population. If passionate supporters of a new party are convinced about its importance, they vote for this new party, although they lose their influence on the composition of parliament in the current election. However, they might influence the results of future elections by pushing more moderate supporters to vote for the new party in future elections.

By extending the time horizon of voters' perceptions, we offer a strategic explanation for a voting behavior that looks wasteful at first sight. The explanation is also valid in situations when sincere voting is not strategically optimal.

Our results suggest some paths for future research. While we concentrate on election thresholds in a proportional system, the basic intuition carries over to majoritarian systems. There are differences, though, as the effective election threshold in a majoritarian system is an endogenous object equal to the minimum number of votes to secure a majority. Including the possibility to abstain from voting might prove interesting as well. In such an extension, the emerging new party might get initial support from former non-voters. The idea would be that non-voters did not vote because they are more or less indifferent between the incumbent parties. Voting for the new party is thus less costly for them even if the party fails the threshold. In this respect, such voters mirror the preferences of the passionates types in our model. Another field in which the basic result of our model might be applied are the dynamics of civil protests. Starting a civil protest can involve high personal cost, but yields the prospect of others joining in once a movement has gained a critical mass. Meirowitz and Tucker (2013) and Kricheli, Livne, and Magaloni (2011) analyze these kinds of scenarios, albeit with quite a different modeling approach.

A. Appendix

Proof of Lemma 1

If either L or R has strictly more than 50% of seats, this party implements its peak. If neither has an outright majority, then a coalition of any two parties holds at least 50% of seats. Suppose party J is the proposer and proposes coalition C. If it makes an offer that is not accepted, x_d is implemented and no spoils are distributed. Such an outcome is dominated by proposing x_d alongside an equal share of S to all parties. Thus, we are looking for the best offer that is accepted. Obviously, party J always proposes $s_I = 0$ to any party I outside the coalition C. Then the optimal proposal of party J for coalition C solves

$$\max_{a,b,s_J,s_{-J}} u^J(a,b) + s_J$$
$$\text{subject to } u^K(a,b) + s_K \geq u^K(x_d) \ \forall K \in C$$
$$\text{and } \sum_{K \in C} s_K \leq S.$$

Note that the solution of that problem maximizes $\sum_{K \in C} u^K$. We calculate the optimal proposal for the three possible proposers.

1. L as proposer:

In a coalition with R, policy $(1/2, 0)$ will be implemented. As this is equal to x_d, L can grab all spoils of office and gets utility $u^L(x_d) + S = -1/4 + S$.
In a coalition with E, policy $x_{LE} = (0, 1/2)$ will be implemented. E likes x_{LE} much better than x_d and is actually willing to "pay" L for offering a coalition. In fact, E is willing to accept $s_E = u^E(x_d) - u^E(x_{LE}) = -1$. Hence L gets utility $u^L(x_{LE}) + S + 1 = 3/4 + S$

which is bigger than its utility from a coalition with R.

If L proposes a *grand coalition* containing all three parties, the value-maximizing policy is $x_{LRE} = (1/3, 1/3)$. Furthermore, the necessary payments to coalition members are $s_R = u^R(x_d) - u^R(x_{LRE}) = 11/36$ and $s_E = u^E(x_d) - u^E(x_{LRE}) = -25/36$. Thus, L receives utility $u^L(x_{LRE}) + S - 11/36 + 25/36 = S + 1/6$ which is smaller than its utility from a coalition with E only. If L has exactly 50% of seats, it could propose a single-party government and receive $0 + S$ if L is chosen as proposer. L prefers, however, to propose a coalition with E and policy $x_{LE} = (0, 1/2)$.

2. R proposes a coalition with E and a policy offer $x_{RE} = (1/2, 1/2)$, as

$$U^R = \begin{cases} u^R(x_{RE}) + u^E(x_{RE}) - u^E(x_d) + S = 1/4 + S & \text{in a coalition with } E \\ u^R(x_{RL}) + u^L(x_{RL}) - u^L(x_d) + S = -1/4 + S & \text{in a coalition with } L \\ u^R(x_{RLE}) + u^L(x_{RLE}) - u^L(x_d) & \\ \qquad +u^E(x_{RLE}) - u^E(x_d) + S = 1/6 + S & \text{in a grand coalition} \\ u^R(x_R) = 0 + S & \text{in a single-party government.} \end{cases}$$

3. E proposes a coalition with L and a policy offer $x_{LE} = (0, 1/2)$, as

$$U^E = \begin{cases} u^E(x_{LE}) + u^L(x_{LE}) - u^L(x_d) + S = -1/4 + S & \text{in a coalition with } L \\ u^E(x_{RE}) + u^R(x_{RE}) - u^R(x_d) + S = -3/4 + S & \text{in a coalition with } R \\ u^E(x_{RLE}) + u^L(x_{RLE}) - u^L(x_d) & \\ \qquad +u^R(x_{RLE}) - u^R(x_d) + S = -5/6 + S & \text{in a grand coalition.} \end{cases} \quad \square$$

Proof of Lemma 2

We only have to consider the case that no party has an absolute majority. Given a coalition C, the implemented policy just depends on the members of the coalition C and remains unchanged from Lemma 1. Differing the default policy only changes the distribution of the spoils of office. Denote the default policy x_d by (a, b). Again, we go through the cases of the three possible proposers.

1. L proposes LE if $b \leq a$ and $a^2 + b^2 \leq -1/6 + 2a$, as

$$U^L = \begin{cases} -1/2 + S + a^2 + (1-b)^2 & \text{in a coalition with } E \\ -1/2 + S + (1-a)^2 + b^2 & \text{in a coalition with } R \\ -4/3 + S + a^2 + (1-a)^2 + b^2 + (1-b)^2 & \text{in a grand coalition.} \end{cases}$$

2. R proposes RE if $b \leq 1/4$ and $a^2 + b^2 \leq 1/3$, as

$$U^R = \begin{cases} -1 + S + a^2 + (1-b)^2 & \text{in a coalition with } E \\ -1/2 + S + a^2 + b^2 & \text{in a coalition with } L \\ -4/3 + S + 2a^2 + b^2 + (1-b)^2 & \text{in a grand coalition.} \end{cases}$$

3. E proposes LE if $a \geq 1/4$ and $a^2 + b^2 \leq -1/6 + 2a$, as

$$U^E = \begin{cases} -1 + S + (1-a)^2 + b^2 & \text{in a coalition with } R \\ -1/2 + S + a^2 + b^2 & \text{in a coalition with } L \\ -4/3 + S + a^2 + (1-a)^2 + 2b^2 & \text{in a grand coalition.} \end{cases}$$

In summary, all the conditions are satisfied if $b \leq 1/4 \leq a$ and $a^2 + b^2 \leq 1/3$. □

Proof of Proposition 1

Using backward induction, we begin with the second period.

Period 2: First, let party E's vote share in period one be small, i. e., $v_E^1 < e$. According to the equilibrium strategies, $v^2 = (v_R^2, v_L^2, v_E^2) = (n_r, n - n_r, 0)$. Then, given Assumption 1, L alone forms government, and implements $x_L = (0, 0)$. An individual deviation only makes a difference if $n_R + 1 \geq n - n_R$. In that case, left-leaning voters strictly prefer voting for L, while right-leaning voters cannot change the result, as already all of them have voted for R. The conditional second-period payoffs are

$$\mathbb{E}(u(X^2;\theta)|v_E^1 < e;\theta) = u(0,0;\theta) = \begin{cases} -\frac{1}{2} & \text{for } \theta = \theta^R \\ 0 & \text{for } \theta = \theta^L \\ \alpha^P - 1 & \text{for } \theta = \theta^P \\ \alpha^M - 1 & \text{for } \theta = \theta^M. \end{cases}$$

Second, consider $v_E^1 \geq e$. On the equilibrium path, $v^2 = (n_r, n - n_r - N_M - N_P, N_M + N_P)$. Coalition bargaining yields equilibrium outcomes as described in Lemma 1. Hence, with probability $v_R^2/n = n_R/n =: r$ party R is chosen to form a coalition and the policy is x_{RE}, whereas with probability $1-r$ the proposer is party L or E and the policy is x_{LE}. Note that, by Assumption 1 and the definition of e, no party gains an absolute majority.

Deviating is unprofitable, because it either has no effect on policy (change between L and E) or increases the chances of the less-preferred coalition. Expected utility is

$$\mathbb{E}(u(X^2;\theta)|v_E^1 \geq e;\theta) = \begin{cases} -r\frac{1}{4} - (1-r)\frac{5}{8} = \frac{1}{8}(3r-5) & \text{for } \theta = \theta^R \\ -r\frac{1}{4} - (1-r)\frac{1}{8} = -\frac{1}{8}(1+r) & \text{for } \theta = \theta^L \\ -r\frac{1}{4} - (1-r)\frac{1}{4}(1-\alpha^P) = -\frac{1}{4}(1-\alpha^P(1-r)) & \text{for } \theta = \theta^P \\ -r\frac{1}{4} - (1-r)\frac{1}{4}(1-\alpha^M) = -\frac{1}{4}(1-\alpha^M(1-r)) & \text{for } \theta = \theta^M. \end{cases}$$

Period 1: Given equilibrium strategies and Assumption 1, only L and R enter parliament and either x_L or x_R is implemented depending on the vote shares. E does not enter parliament, even if an individual voter switches her vote to E.

Voters may have incentives to deviate in order to change the probabilities of policies x_L and x_R (*short-term incentive*) or in order to change voting behavior in period 2 (*long-term incentive*). In the following, we rule out any one-stage deviation.

Incentives of a left type:
Switching to R is unprofitable; it has a negative short-term effect and no long-term effect. Switching to E has a negative short-term effect as well. Furthermore, it increases the chance of meeting threshold e. Left types, however, prefer the very opposite with respect to period 2.

Incentives of a right type:
A vote for L has a negative short-term effect (compared to voting for R) and there is no long-run effect.

Voting for E instead of R can be pivotal in two respects: In the short run, it might lead to a left government rather than a right one. In the long run, it could induce a coalition government rather then a left one. The total change in utility is

$$\begin{aligned} dU &= \Pr\left[N_M + N_L \in \{n_R - 1, n_R\} | \theta = \theta^R\right] \frac{u(x_L;\theta^R) - u(x_R;\theta^R)}{2} \\ &\quad + \delta \Pr\left[N_P + 1 = e | \theta = \theta^R\right] \left(\mathbb{E}(u(X^2;\theta^R)|v_E^1 \geq e;\theta^R) - \mathbb{E}(u(X^2;\theta^R)|v_E^1 < e;\theta^R)\right) \\ &= -\frac{1}{4}\Pr\left[N_M + N_L \in \{n_R - 1, n_R\} | \theta = \theta^R\right] + \delta \Pr\left[N_P + 1 = e | \theta = \theta^R\right] \frac{3r-1}{8} \end{aligned}$$

The deviation is not profitable if

$$2\Pr\left[N_M + N_L \in \{n_R - 1, n_R\} | \theta = \theta^R\right] \geq \delta \Pr\left[N_P = \tau - \underline{n_M} | \theta = \theta^R\right](3r-1)$$

Assumption 2 ensures this condition.

Incentives of a moderate type:
Switching the vote from L to R is clearly not profitable. Switching to E lowers short-term expected utility, but may help E to enter parliament later, if it triggers coordinated voting for E in period 2. This is unprofitable if the short-term losses

$$\left(\Pr\left[N_L + N_M = n_R|\theta^M\right] + \Pr\left[N_L + N_M = n_R + 1|\theta^M\right]\right)\frac{u(x_R;\theta^M) - u(x_L;\theta^M)}{2} =$$
$$= -\left(\Pr\left[N_L + N_M = n_R|\theta^M\right] + \Pr\left[N_L + N_M = n_R + 1|\theta^M\right]\right)\frac{\alpha^M}{2}$$

are higher than the long-term gains from switching

$$\delta\Pr\left[N_P = \tau - \underline{n_M}|\theta^M\right]\left(\mathbb{E}(u(X^2;\theta^M)|v_E^1 \geq \tau + 1 - \underline{n_M};\theta^M) - u(x_L;\theta^M)\right) =$$
$$=\delta\Pr\left[N_P = \tau - \underline{n_M}|\theta^M\right]\frac{1}{4}\left(3 - \alpha^M(r+3)\right).$$

This requires

$$\alpha^M \geq \frac{3\delta\Pr\left[N_P = \tau - \underline{n_M}|\theta^M\right]}{2\Pr\left[F_1\right] + \delta\Pr\left[N_P = \tau - \underline{n_M}|\theta^M\right](3+r)} \tag{4}$$

with the event F_1 defined by $N_L + N_M \in \{n_R, n_R + 1\}$.
Incentives of a passionate type:
Passionate types have a short term incentive to switch their vote to L to increase the probability of L getting the majority. On the other hand, switching to L reduces the chances that $v_E^1 \geq \tau + 1 - \underline{n_M}$, thus decreasing the probability of E entering parliament in period 2. The short-term benefits from switching are:

$$\left(\Pr\left[N_L + N_M = n_R|\theta^P\right] + \Pr\left[N_L + N_M + 1 = n_R|\theta^P\right]\right)\frac{u(x_L;\theta^P) - u(x_R;\theta^P)}{2} =$$
$$= \Pr\left[F_2|\theta^P\right]\frac{\alpha^P}{2}$$

with the event F_2 defined by $N_L + N_M \in \{n_R - 1, n_R\}$. The long-term losses from switching are:

$$-\delta\Pr\left[N_P = \tau + 1 - \underline{n_M}|\theta^P\right]\left(\mathbb{E}(u(X^2;\theta^P)|v_E^1 \geq \tau + 1 - \underline{n_M};\theta^P) - u(x_L;\theta^P)\right) =$$
$$= -\delta\Pr\left[N_P = \tau + 1 - \underline{n_M}|\theta^P\right]\frac{1}{4}\left(3 - \alpha^P(r+3)\right)$$

We find that the deviation is unprofitable if $\Pr[F_2|\theta^P] = 0$ or

$$\alpha^P \leq \frac{3\delta \Pr\left[N_P = \tau + 1 - \underline{n_M}|\theta^P\right]}{2\Pr\left[F_2|\theta^P\right] + \delta \Pr\left[N_P = \tau + 1 - \underline{n_M}|\theta^P\right](r+3)}. \tag{5}$$

Switching the vote from E to R is unprofitable; it has a negative short-term effect, and the long-term effect is negative if $\alpha^P \leq 3/(3+r)$, which is implied by (5).

Note that conditions (4) and (5) are compatible with $0 \leq \alpha^P < \alpha^M < 1$. □

Proof of Corollary 1

Corollary 1 follows immediately from the statement of Proposition 1. □

Proof of Proposition 2

The proof proceeds analogously to the proof of Proposition 1. Only the incentives of a right type change.

A vote for L has a negative short-term effect (compared to voting for R). It suffices to show that the long-run effect is non-positive:

$r = n_R/n > 1/3$: Voting for L increases the threshold e and reduces the probability of E entering parliament. Payoff $\mathbb{E}(u(X^2;\theta^R)|v_E^1 < e;\theta^R) = -1/2$ becomes more likely, whereas payoff $\mathbb{E}(u(X^2;\theta^R)|v_E^1 \geq e;\theta^R) = (3r-5)/8$ becomes less likely. Yet the R type voter does not profit from the deviation, as $-1/2 < (3r-5)/8 \Leftrightarrow r > 1/3$.

$r \leq 1/3$: In this case, switching has no long-run effect, as the threshold e is not affected.

Voting for E instead of R has a negative short-term effect. It suffices to show that the long-run effect is non-positive.

$r > 1/3$: Voting for E not only increases the number of votes E receives, but also the threshold e. Consequently, the probability of E meeting the threshold does not change. Hence switching has no long-term effect.

$r \leq 1/3$: In this case, switching does increase the probability that E meets the threshold. Payoff $\mathbb{E}(u(X^2;\theta^R)|v_E^1 < e;\theta^R) = -1/2$ becomes less likely, whereas payoff $\mathbb{E}(u(X^2;\theta^R)|v_E^1 \geq e;\theta^R) = (3r-5)/8$ becomes more likely. Yet the R type voter does not profit from the deviation as $-1/2 \geq (3r-5)/8 \Leftrightarrow r \leq 1/3$.

Hence, there is no profitable deviation for a right type. □

Proof of Proposition 3

Suppose everyone is voting sincerely in both periods. Now, consider the following deviation. A voter of type θ^M votes sincerely in the first period. If the entrant receives less than τ votes in the first period, she votes for L instead of E in the second period. This deviation could make L win the election instead of R increasing the voter's utility by α^M.

After the first election, voters learn the sum of passionate θ^P and moderate types θ^M. Assumption 1 guarantees that with positive probability the sum of passionate θ^P and moderate types θ^M is smaller than the election threshold τ. In addition, with positive probability the number of left types θ^L equals n_R or $n_R - 1$. Finally, both events together occur with positive probability. Hence, the deviation is profitable in expectation. Consequently, voting sincerely in both periods is not an equilibrium. □

Proof of Proposition 4

First, notice that the set of priors consistent with Assumptions 1 and 3 is non-empty.

Now, consider a voter of type θ^M in period one. Everyone else votes sincerely. By deviating from sincere voting, the voter may be pivotal in two respects. First, switching to L may help L to win a majority if E does not enter parliament. Second, switching may make E fail the threshold τ in period one, changing period-one government and implying a left government in period two instead of a three-party parliament. To simplify notation, let $u_K = u(x_K;\theta^M)$ for $K = L, R$ and let $u_E = ru(x_{RE};\theta^M)+(1-r)u(x_{LE};\theta^M)$ (with $x_{(\cdot)}$ as given in Lemma 1). Then, by deviating to L instead of E in the first period, the moderate's utility changes by

$$\begin{aligned}
dU = {} & \Pr[N_M + N_P = \tau \wedge N_L + 1 < n_R|\theta^M]\,(u_R - u_E + \delta(u_L - u_E)) \\
& + \Pr[N_M + N_P = \tau \wedge N_L + 1 = n_R|\theta^M]\,((u_R + u_L)/2 - u_E + \delta(u_L - u_E)) \\
& + \Pr[N_M + N_P = \tau \wedge N_L + 1 > n_R|\theta^M]\,(u_L - u_E + \delta(u_L - u_E)) \\
& + \Pr[N_M + N_P < \tau \wedge N_L + 1 = n_R|\theta^M]\,((u_R + u_L)/2 - u_R) \\
& + \Pr[N_M + N_P < \tau \wedge N_L = n_R|\theta^M]\,(u_L - (u_R + u_L)/2) \\
= {} & \Pr[N_M + N_P = \tau \wedge N_L + 1 < n_R|\theta^M]\,(u_R - u_E + \delta(u_L - u_E)) \\
& + \Pr[N_M + N_P = \tau \wedge N_L + 1 = n_R|\theta^M]\,((u_R + u_L)/2 - u_E + \delta(u_L - u_E)) \\
& + \Pr[N_M + N_P = \tau \wedge N_L + 1 > n_R|\theta^M]\,(u_L - u_E + \delta(u_L - u_E)) \\
& + \Pr[N_M + N_P < \tau \wedge N_L \in \{n_r - 1, n_R\}|\theta^M]\,((u_L - u_R)/2)
\end{aligned}$$

$$
\begin{aligned}
&= \Pr[N_M + N_P = \tau|\theta^M](\delta u_L - (1+\delta)u_E) \\
&\quad + u_R\Big(\Pr[N_M + N_P = \tau \wedge N_L + 1 < n_R|\theta^M] + \frac{1}{2}\Pr[N_M + N_P = \tau \wedge N_L + 1 = n_R|\theta^M] \\
&\qquad - \frac{1}{2}\Pr[N_M + N_P < \tau \wedge N_L \in \{n_r - 1, n_R\}|\theta^M]\Big) \\
&\quad + u_L\Big(\frac{1}{2}\Pr[N_M + N_P = \tau \wedge N_L + 1 = n_R|\theta^M] + \Pr[N_M + N_P = \tau \wedge N_L + 1 > n_R|\theta^M] \\
&\qquad + \frac{1}{2}\Pr[N_M + N_P < \tau \wedge N_L \in \{n_r - 1, n_R\}|\theta^M]\Big) \\
&= \Pr[N_M + N_P = \tau|\theta^M](\delta u_L - (1+\delta)u_E) + u_R\big(\Pr[N_M + N_P = \tau|\theta^M] - P^L\big) + u_L P^L \\
&= P^L(u_L - u_R) + \Pr[N_M + N_P = \tau|\theta^M](u_R - u_E + \delta(u_L - u_E)).
\end{aligned}
$$

We plug in $u_L = \alpha^M - 1$, $u_R = -1$ and $u_E = -(1 - \alpha^M(1-r))/4$ and rearrange into

$$
\begin{aligned}
dU &= \alpha^M(T\delta + P^L) - \alpha^M(1-r)T(1+\delta)/4 - 3T(1+\delta)/4 \\
&= \alpha^M\Big(T\delta(3/4 + r/4) + P^L - (1-r)T/4\Big) - 3T(1+\delta)/4
\end{aligned}
$$

with $T = \Pr[N_M + N_P = \tau|\theta^M]$. Then dU is greater 0 if

$$
\alpha^M > \frac{3T(1+\delta)}{4P^L - (1-r)T + \delta(3+r)T} \tag{6}
$$

Assumption 3 ensures that the fraction (6) is smaller than one. To sum up, the deviation to L is profitable for the moderate type, if inequality (6) is satisfied. □

Proof of Corollary 2

The statement of Corollary 2 follows directly from Proposition 4 with $\delta = 0$. □

Proof of Lemma 3

For every prior consistent with Assumption 3, condition (6) is feasible. In particular, $\alpha^M = 1$ ensures condition (6). In addition, for every prior consistent with Assumption 1, there are α^P and α^M with $\alpha^P < \alpha^M$ that satisfy conditions (4) and (5). In particular, $\alpha^P = 0$ and $\alpha^M = 1$ guarantee both conditions. □

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